FAIL-SAFE CHARACTERISTICS OF BUILT-UP SHEET STRUCTURES *

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Abstract

During the operational life of an aircraft structure cracks or partial failures may occur. When it concerns a fail-safe design the airworthiness requirements demand that the structure can still withstand a prescribed load when a certain amount of damage is present. It is essential then that the damage can be detected during regular inspections before it has extended to a dangerous size. Thus, apart from reliable inspection procedures, a thorough knowledge of crack propagation and residual strength characteristics of fail-safe structures is required.

Much data can be found in the literature about residual strength and crack propagation of unstiffened cracked sheets. However, aircraft structures consist largely of built-up sheet structures and far less information is available about this structural configuration. Much effort has been devoted to these problems by NLR.

The present paper presents some results of residual strength and crack propagation computations for stiffened panels, using unstiffened panel data and accounting for the sheet-stiffener interaction. The computational results are compared with experimental data.

1. Introduction

The major part of most aircraft structures consists of plain sheet or built-up sheet structures. During the operational life of the aircraft cracks or partial failures may arise in these structural elements. These cracks may extend to a considerable size, due to service loadings. Safety requires, however, that cracks can be detected, during regular inspections, before they have attained a dangerous size. It also requires a structural design that can still withstand a prescribed load when a certain amount of damage is present. The structure that meets these requirements is called fail-safe.

The various aspects of the fail-safe problem are illustrated in Fig.1 for an unstiffened sheet structure. After a certain period in service a crack may initiate (point A_o in upper diagram). Initially this crack will be too small to be detected by any of the existing inspection techniques. After some time (A hours) it has grown to a size that allows detection. With increasing crack length the remaining strength of the structure (= residual strength) will gradually decrease and after B hours it will drop below the required fail-safe strength (see lower diagram).
Apparently the period from A to B is available for crack detection. For a safe operation there should be at least two or three inspections in this period to prevent that a crack of the minimum detectable size, that just escaped attention during the last inspection, will grow to a critical size before the following inspection can be made.

It follows that for a given inspection technique the residual strength and crack propagation characteristics of the structure are of paramount importance for the fail-safety of the aircraft. The prediction of these characteristics becomes more complex for a sheet structure with stiffeners. Both the crack propagation rate and the residual strength will be affected favourably by the stiffeners. Further, in a proper fail-safe design the stiffeners can act as crack stoppers in the case of fracture instability. On the other hand, however, in a cracked structure load will be transferred to the stiffeners leading to a shorter fatigue life of the stiffeners. This is of special importance if a crack is either initiated or arrested under a stif-

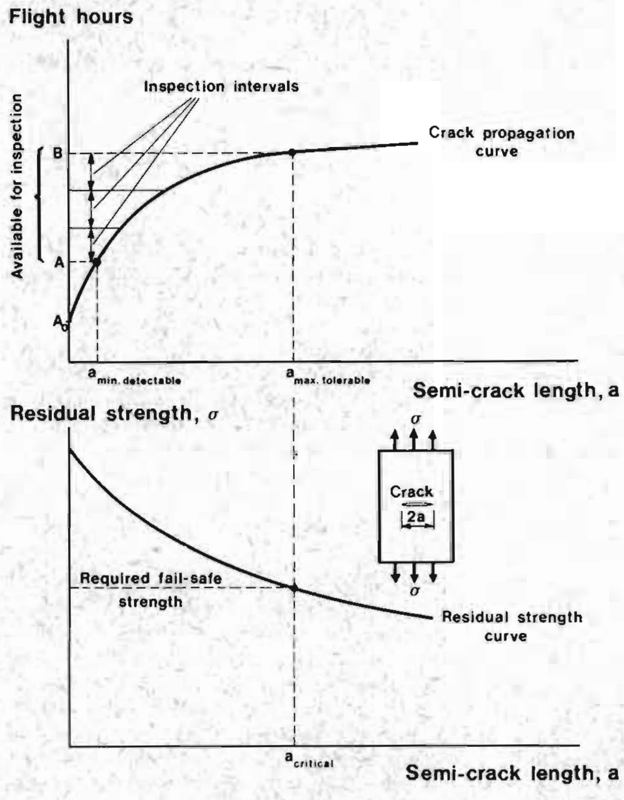

Fig.1 Aspects of fail-safe problem.

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fener. Especially in the latter case failure of the stiffener can lead to a dangerous situation.

In a proper fail-safe design analysis all the aforementioned aspects have to be accounted for. The NLR is devoting much effort to the fail-safe problem. Presently an analytical computer program is available that enables the prediction of the residual strength and fatigue crack propagation characteristics for a stiffened structure, starting from the strength and fatigue properties of sheet and stiffener separately. The analysis is accounting for the sheet-stiffener interaction. The aspects of crack arrest and stiffener failure (due to peak loads or fatigue) are incorporated. In the following sections the details of the analysis will be dealt with. The residual strength and crack propagation problem will first be discussed separately. Then the results will be combined in order to discuss the various aspects of the fail-safe problem of a stiffened structure.

2. Interaction of sheet and stiffeners

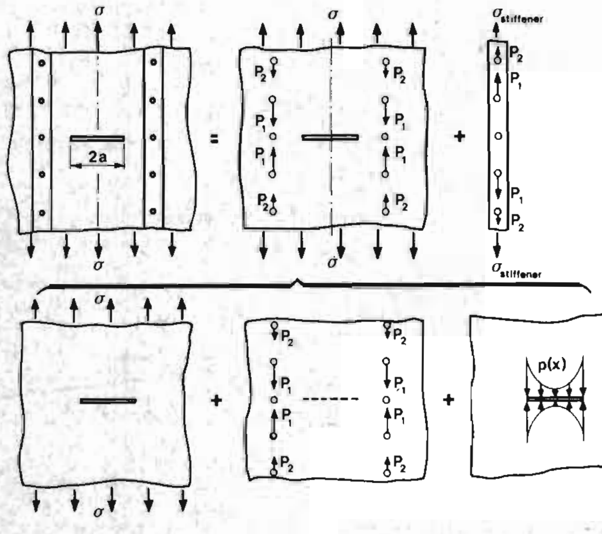
It can be shown by the theory of elasticity that the stresses at the tip of a crack in a sheet are determined by the stress intensity factor, K, defined by

$$K = \alpha \cdot \sigma\sqrt{\pi a} \qquad (1)$$

In this expression σ is the gross stress remote from the crack, a is the semi-crack length and α is a factor accounting for limited panel size. Generally, cracks in aircraft structures are limited to a small fraction of the panel width, implying that the correction factor is close to 1. For this reason the factor α will be omitted henceforth in the expressions of the stress intensity factor.

Fracture mechanics assumes that both crack growth rate and residual strength are governed by the value of the stress intensity factor. In the case of a cracked stiffened panel the stiffening elements provide extra stiffness to the cracked sheet. In the region of the crack the stiffeners take over some load from the skin (see upper half of Fig.2), such that the stress intensity factor in the stiffened panel is lower than in the unstiffened panel with the same length of crack. The effect of the stiffener on the stress condition at the crack tip is expressed by the "tip stress correction factor", C (C < 1). The stress intensity factor of the stiffened panel can then be given as

$$K = C\,(a) \cdot \sigma\sqrt{\pi a} \qquad (2)$$



p(x) is stress distribution along x-axis due to fastener loads P_i

Fig.2 Break down of cracked stiffened panel into its components to compute fastener loads P_i.

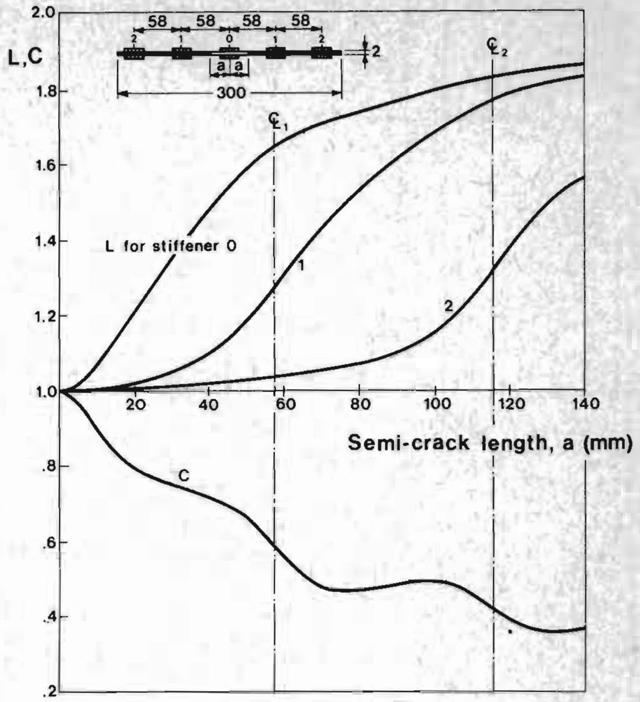


Fig.3a Values of L and C for a riveted panel with five intact strip stiffeners. Total stiffener cross-sectional area is 50 per cent of total gross area. Crack passes through rivet holes.

On the other hand the presence of the crack in the stiffened panel will locally enforce a higher load in the stiffeners and in the fasteners. The overload of the stiffener due to the crack is expressed by the value of the "stiffener load concentration factor", L (L > 1). The maximum load in the stiffener (F_{max}) occurs in the region of the crack and will be given by (see upper half of Fig.2)

$$F_{max} = \sigma_{stiffener}\, A_s + \sum_{i=1}^{n} P_i = L(a)\, \sigma_{stiffener}\, A_s \qquad (3)$$

where $\sigma_{stiffener}$ is the stiffener stress at the loaded end of the panel, A_s is the stiffener cross-sectional area and P_i are the fastener loads. If there is a uniform stress distribution in the panel remote from the crack, then $\sigma_{stiffener}$ equals the nominal stress σ in the sheet.

It has to be remarked here that the load increase in the stiffener may become so large that fracture of the stiffener occurs. In that case the load of the broken stiffener will be transmitted to the sheet so that the stringer has an adverse effect on the stress intensity factor.

Apparently C and L are important parameters to determine the stiffener effectiveness with respect

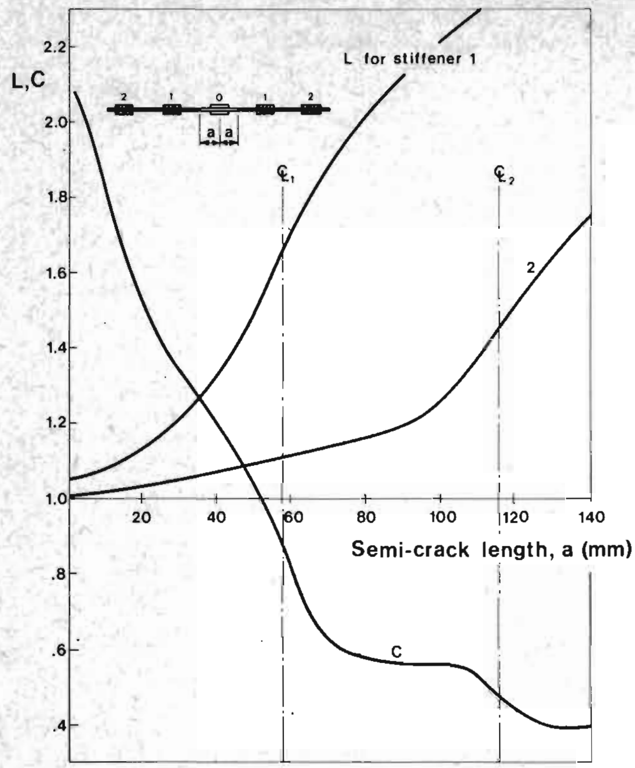


Fig.3b Values of L and C for the panel configuration of figure 3a but with a broken central stiffener.

to crack growth and residual strength. The values of C and L can be readily calculated when the fastener loads are known. The procedure of calculating the fastener loads in a cracked stiffened panel is illustrated in Fig.2 for a panel with edge stiffeners and a crack in between. The calculation is based on the condition of equal displacements of the corresponding fastener points of sheet and stiffener. The stiffened structure is split up into parts as depicted in the upper half of Fig.2. The displacement in the cracked sheet is a superposition of three components indicated in the lower half of Fig.2. Full details about this method of computating the fastener loads are given in Ref.1. When the fastener loads are known the value of L can be determined from equation (3), while the value of C can be computed from

$$K = \left(\sigma - \frac{2}{\pi}\int_0^a \frac{p(x)\,dx}{\sqrt{a^2 - x^2}}\right)\sqrt{\pi a} \qquad (4)$$

where p(x) is the stress distribution along the x-axis due to the opposing rivet forces. Values of C and L obtained in this manner are given in Fig.3 for a panel configuration with five symmetrical stiffeners and a central crack. Fig.3a applies to a panel with an intact central stiffener whereas in Fig.3b the effect of a combination of a skin crack and a broken central stiffener is depicted. As shown by Fig.3a the unbroken central stiffener gives a considerable reduction of the tip stress. However, a broken stiffener increases the tip stress most significantly (see Fig.3b). Obviously the load concentration in the adjacent stiffeners is also more severe in the latter case.

3. Residual strength characteristics of stiffened panels

3.1 Unstiffened panel behaviour

The behaviour of an unstiffened cracked panel with an initial crack $2a_o$, subjected to an increasing stress, is depicted in Fig.4. The crack will start to extend slowly at a stress level σ_i. Initially the crack will grow in a stable way, because crack growth will proceed only when the stress is further raised. Unstable crack growth will occur at a stress level σ_c. The crack then propagates extremely fast through the sheet, resulting in complete fracture. Both slow stable crack growth and fast fracture instability occur at lower stress levels if the initial crack is longer. By testing panels with different initial crack sizes the curves of Fig.4 can be obtained. The solid curve gives the stress that can be carried by the panel when a crack of a certain size is present. This curve is the residual strength curve of the unstiffened panel.

3.2 Stiffened panel behaviour

If the cracked sheet is provided with stiffeners the effect of the stiffening will be a reduction of the stress intensity factor and an increase of the stiffener load in the region of the crack. The effect of this sheet-stiffener interaction on the shape of the residual strength diagram of Fig.4 is illustrated in Fig.5. For simplicity the cracked sheet is provided with only two stiffeners.

Assume that crack propagation in the stiffened panel occurs at the same stress intensity as in the unstiffened panel. From equation (2) and Fig.3a it can be concluded then that crack propagation in the stiffened panel will occur at a higher stress level than in the unstiffened panel, the increase being dependent upon crack size. This means that the

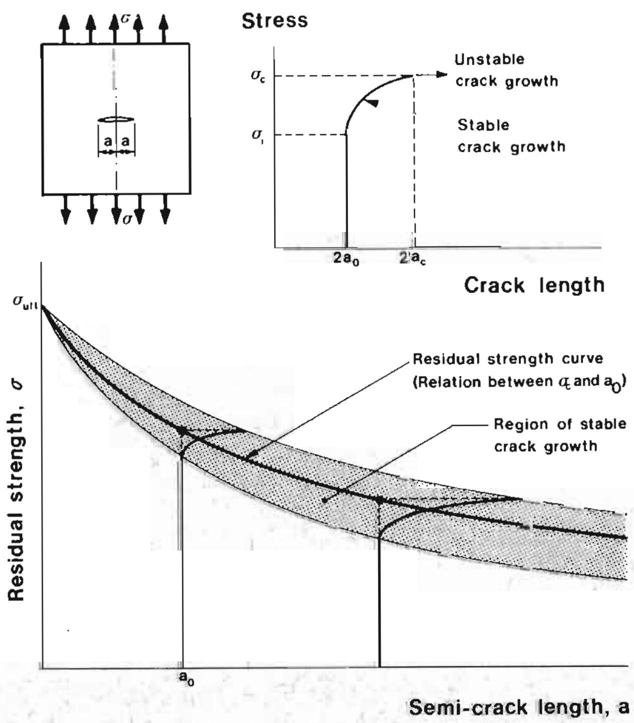


Fig.4 Residual strength diagram of unstiffened panel.

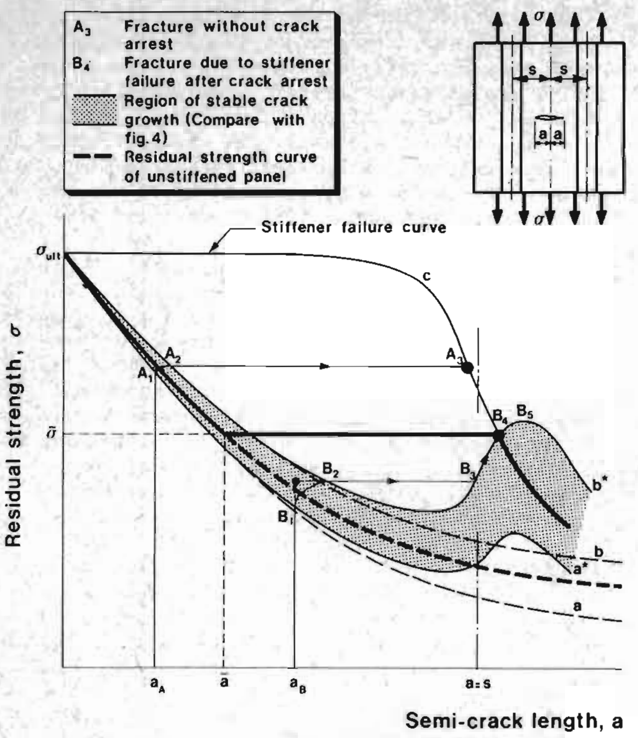


Fig.5 Residual strength diagram of stiffened panel.

curves a and b of the unstiffened panel will be shifted upwards to a^* and b^*, respectively. Curves a^* and b^* show a maximum for a crack slightly larger than the stringer spacing because the maximum reduction in tip stress will occur when the crack tip has just passed the stiffener center line (see Fig.3a). Consequently the region of stable crack growth of the stiffened panel will have the shape as indicated by the shaded area. However, in a stiffened panel the possibility of stiffener failure should also be considered. Curve c in Fig.5 is the locus for stiffener failure. At zero crack length the stiffener will fail at its ultimate tensile strength. With increasing crack size the load concentration in the stiffener will increase, implying that a lower failure stress at the panel is required.

Now consider the behaviour of the stiffened panel for two different crack lengths. When the panel contains a crack that is small as compared to the stiffener spacing (semi-crack length a_A in Fig.5) the stress condition at the crack tip will hardly be influenced by the presence of the stiffeners. Stable crack growth and fracture instability will start at the same stress levels as in the unstiffened sheet (points A_1 and A_2). When the unstably growing crack approaches the stiffener, the load concentration in the stiffener will be so high that the stiffener fails without stopping the unstable crack growth (point A_3).
If the initial crack length is larger, a_B, unstable crack growth will occur at a stress slightly above the fracture strength of the unstiffened sheet (point B_2), but this crack will be stopped under the stiffeners (point B_3) due to the tip stress reduction. After crack arrest the load on the panel can be further increased, the tip stress is raised and some additional stable crack growth occurs before the ultimate stringer load is reached (point B_4). For any initial crack length between $\bar{a}$ and s the behaviour will be essentially the same as sketched for crack length a_B, fracture always occurring at the stress level indicated by $\bar{\sigma}$.
This implies a predicted residual strength curve of the shape heavily drawn in Fig.5. The curve contains a horizontal part determined by the stiffener strength and sheet crack resistance and by the relative stiffness of sheet and stiffener (Ref.2). For initial crack lengths smaller than the stiffener spacing this flat part of the curve constitutes a lower bound of the residual strength of the stiffened panel and, hence, $\bar{\sigma}$ will be the fail-safe stress of the stiffened panel.

It should be pointed out here that the stiffener failure curve in Fig.5 intersects the stiffened sheet crack resistance curve (= curve that relates σ_c and a_c of stiffened sheet). In that case failure of the panel will occur due to stiffener failure. However, the stiffener failure curve does not necessarily intersect the sheet crack resistance curve. If the curves do not intersect, failure of the panel will occur, after crack arrest, by sheet failure at point B_5 in Fig.5.

It can be concluded from the foregoing that the determination of the stiffened sheet crack resistance curve and of the stiffener failure curve is essential in predicting the residual strength of a certain stiffened panel configuration. The calculation of the two curves from unstiffened sheet and stiffener properties, by accounting for sheet-stiffener interaction, will be dealt with in the following section.

3.3 Prediction of the residual strength diagram

As shown previously the stress intensity factor of the stiffened sheet can be expressed by (equation 2)

$$K_{stiffened} = C\,(a)\ \sigma\sqrt{\pi a}$$

Assuming that unstable crack growth occurs when $K_{stiffened}$ has a value equal to the plane stress fracture toughness of the unstiffened sheet, K_c, then the stiffened sheet crack resistance curve is given by the relation

$$\sigma_{sheet} = \frac{K_c}{C(a)\sqrt{\pi a_c}} \qquad (5)$$

It has to be emphasized that this assumption for fast fracture (viz. $K_{stiffened} = K_c$) is not essential for the applicability of the present method. A relation between σ_c and a_c of the unstiffened panel has to be available, but this relation does not need to be dictated by K_c = constant. A simple data-plot, giving the failure stress σ_c of the unstiffened panel as a function of a_c, will suffice. In order to apply this to the stiffened panel another assumption is required then: skin crack propagation in the stiffened panel will occur when the stress intensity factor is the same as in the unstiffened sheet at the particular crack length under consideration. Hence the skin crack criterion can also be given by

$$\sigma_{sheet} = \frac{\sigma_c}{C(a)} \qquad (6)$$

This means that the skin crack resistance curve can be obtained by raising all points of the σ_c - a_c curve of the unstiffened panel by a factor $1/C^c(a)$ pertinent to the particular length of crack.

In general the latter procedure has to be applied for 2024-T3 sheet material for which an actual K_c value usually cannot be determined, except for extremely large panels (Ref.2).

Assuming a uniform stress distribution at the loaded end of the panel, the maximum load in the stiffener according to equation (3) will be

$$F_{max} = L\,(a)\,\sigma\,A_s$$

Failure of the stiffener will occur when the value of F_{max} is equal to the ultimate strength of the stiffener (F_{ult}), or when

$$F_{max} = F_{ult} = \psi\,\sigma_{ult}\,A_s \qquad (7)$$

where σ_{ult} is the ultimate tensile strength of the stiffener material and $\psi \leq 1$ is a factor accounting for load eccentricity and notch effects in the stiffener. Combining equations (3) and (7) yields the following relation for the stiffener failure curve

$$\sigma = \psi\,\frac{\sigma_{ult}}{L(a)} \qquad (8)$$

Using these equations the residual strength of a panel with five riveted strip stiffeners was determined. The panel dimensions were 300 x 560 mm (see Fig.12). The stiffener area was approximately 50 per cent of total gross area. The material of sheet and stiffeners was 7075-T6. Crack initiation was assumed to occur under the central stiffener at a rivet hole. Fig.6a gives the results for the panel with five intact stiffeners, whereas Fig.6b applies to the case of a broken central stiffener. The required relation between σ_c and a_c of the unstiffened, 2 mm thick, 7075-T6 sheet was derived from test results. For intermediate crack sizes the relation appeared to be described by $K_c = 264$ kgmm$^{-3/2}$. For higher stress levels ($>2/3\,\sigma_{ys}$) and for larger crack lengths ($>w/3$) the tangents proposed by Feddersen (Ref.3) were assumed to apply. The stiffened sheet crack resistance curve was obtained using equation (6) and the σ_c versus a_c plot of the unstiffened panel. The stiffener failure curves were obtained using equation (8) in combination with the results of tensile tests on riveted strip stiffener specimens to find the value of the factor ψ (Fig.9).

It has to be noted that a prediction of the residual strength diagram based on the elastic results for C and L of Fig.3a, cannot be made for this panel configuration. Due to the high load concentration in the central stiffener yielding of rivets and stiffener will occur at relatively low external loads. For this reason yielding of rivets and stiffeners had to be incorporated in the analytical computer program. Some simplifying assumptions were made in doing so. Firstly, the rivet load was assumed to remain constant after the rivet yield load was attained. Secondly, the non-linear part of the σ-ε curve of the stiffener material was approached by a broken line. The effect of yielding on C and L at a certain crack length was computed by a stepwise increase of the external load starting with the stress level at which yielding first occurred. After each step the rivet loads and the maximum stiffener loads were considered and, if necessary, adjustment of displacement equations was made. Full detail about this procedure is given in Ref.1.

Although it was assumed that the crack initiated under the central stiffener from a rivet hole, this does not necessarily imply that the crack will grow into a rivet hole of the adjacent stiffeners. The crack can deviate and pass between two rivets. In Figs 6a and 6b the effect of the crack path on both

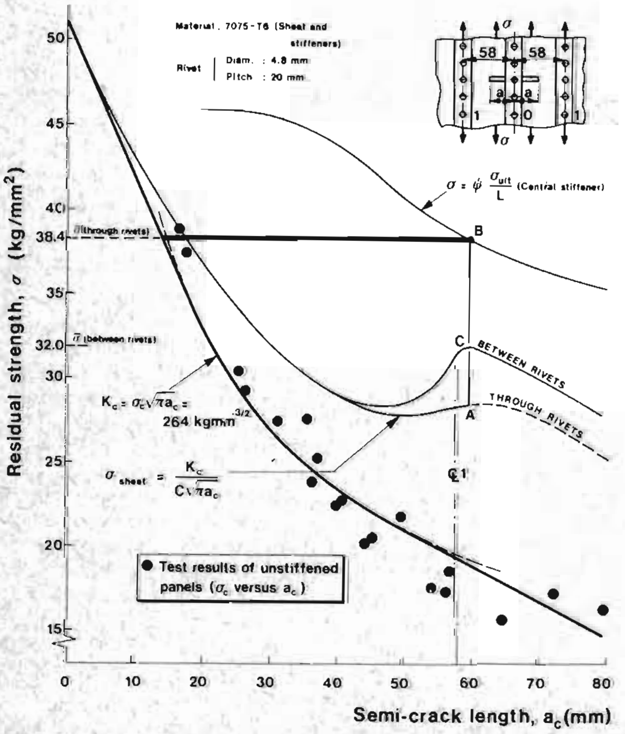


Fig.6a Predicted residual strength diagram for a panel with five riveted strip stiffeners and a central crack.

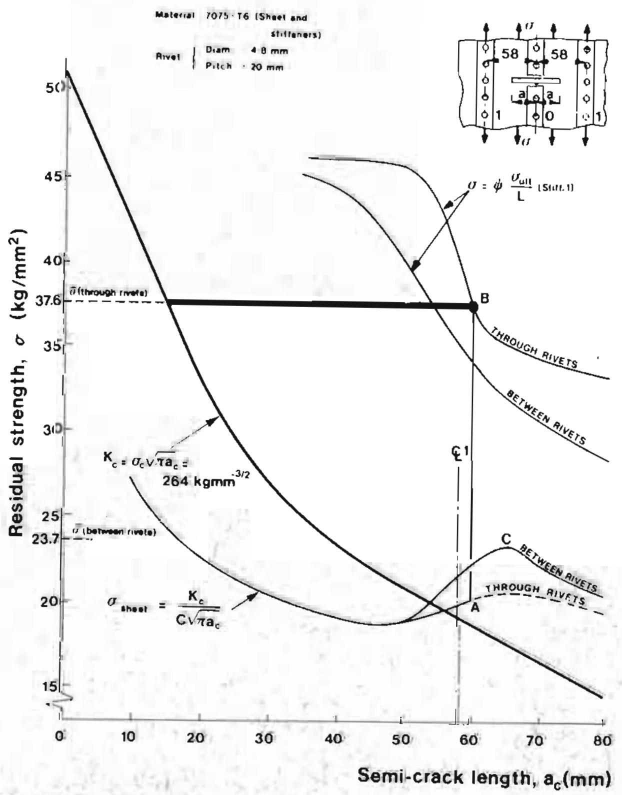


Fig.6b Predicted residual strength diagram for panel configuration of figure 6a but with a broken central stiffener.

sheet crack resistance curve and stiffener failure curve is indicated. However, when the crack runs into a rivet hole there is a benefit of the rivet hole, which has not yet been considered. Now, with increasing external load the semi-crack length will remain equal to the stiffener pitch plus half the rivet diameter due to the blunting effect of the rivet hole. Further crack growth from the rivet hole will occur at a higher stress level than suggested by the sheet crack resistance curve. The stress for further crack growth will be somewhere between points A and B (see Figs 6a and 6b), the delay being dependent upon the size of the hole. In Fig.6 it was assumed that the crack will be arrested in the rivet hole until final failure of the panel occurs due to stiffener failure (point B). In the case the crack passes between two rivets failure of the panel apparently is determined by the top of the relevant sheet crack resistance curve (point C in Fig.6). The resulting fail-safe level for both possible crack paths is indicated on the vertical axes of Figs 6a and 6b.

4. Fatigue crack propagation characteristics of stiffened panels

As shown previously the stress intensity factor is a sufficient parameter to describe the stress field at the tip of a crack. If two different cracks have the same stress environment, i.e. the same stress intensity factor, they may be expected to behave in the same manner under fatigue loading and show the same crack propagation. Fracture mechanics usually assumes that the rate of fatigue crack growth per cycle, da/dn, is a function of the stress intensity factor range during that cycle, ΔK, and of the stress ratio R, being the ratio of the minimum and the maximum stress in a cycle or

$$\frac{da}{dn} = f_R \left[\Delta K\ (a)\right] \qquad (9)$$

where f_R is a function dependent on the stress

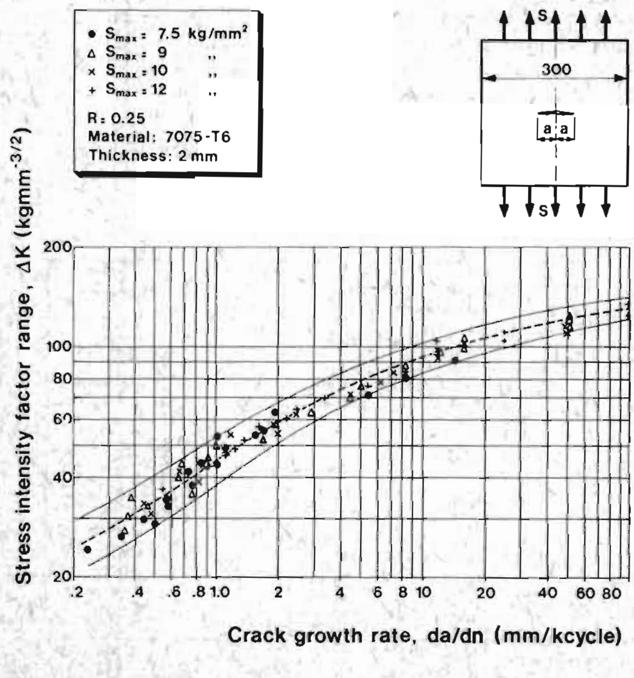


Fig.7 Relation between stress intensity factor range and crack propagation rate of unstiffened sheet.

a (mm)	C (fig. 3a)	ΔK = CΔS√(πa) *)	da/dn (fig. 7) (mm/kcycle)
10(= a_0)	.897	37.69	.61
30	.753	54.68	1.65
50	.670	63.00	2.40
70	.479	53.32	1.48
90	.482	60.72	2.15
110	.466	64.92	2.64
130	.362	54.84	1.60

*) ΔS = 7.5 kg/mm²

Table 1 Computation of crack growth rate for panel with five intact strip stiffeners and a central crack through rivet holes.

ratio. This means that there exists a unique relation between the crack growth rate and ΔK when the stress ratio is a constant. Paris was first to recognize this (Ref.4). If equation (9) were applicable, data obtained from specimens tested at various stress levels but with the same stress ratio should all fall on a single curve. This is confirmed by the plot of constant amplitude test data of 7075-T6 unstiffened sheet material (Fig.7).

In a stiffened panel the stress condition at the crack tip will be affected by the presence of the stiffeners. Their effect on the stress intensity is expressed by the correction factor C (see eq.(2)). This implies that the rate of crack propagation of a stiffened panel can be predicted when the relation between da/dn and ΔK of the unstiffened panel and the dependency of C on crack length are available (Ref.5). Having computed da/dn of the stiffened panel as a function of crack length, the crack propagation curve, giving the relation between n and a, can be obtained by an integration procedure from

$$n = \int_{a_0}^{a_1} \frac{da}{\frac{da}{dn}(a)} \qquad (10)$$

in which a_0 is the initial crack length or the minimum detectable crack length (see Fig.1). In the case of a panel with intact stiffeners and a crack under a stiffener the value of C decreases with increasing crack length (see Fig.3a). This implies a lower stress intensity factor and consequently a retarded crack propagation. Consequently an increase in fatigue life of the stiffened panel compared with that of an unstiffened panel can be expected. However, when considering the fatigue life of a cracked stiffened panel one should also account for the possibility of premature stiffener failure due to fatigue. In a cracked stiffened panel load is transferred from the sheet to the stiffener. This implies that the fatigue life of the stiffener will be reduced compared to that of a stiffener in an uncracked panel, the reduction being dependent on crack length and growth rate. If the stiffener fails then the sheet crack propagation rate will increase rapidly, because the tip stress correction factor C will increase considerably (see Fig.3).

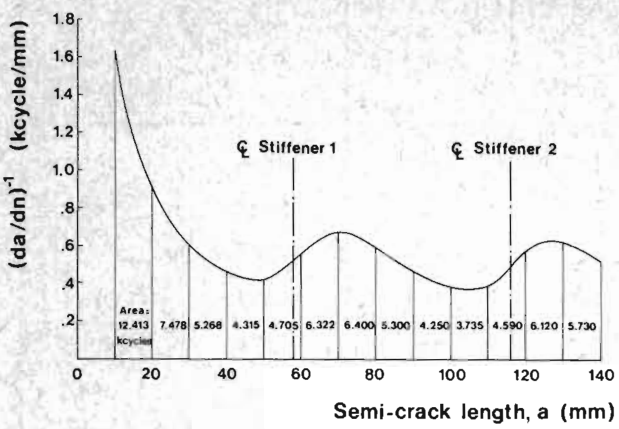


Fig.8 Computation of n using equation (10) and results of table 1.

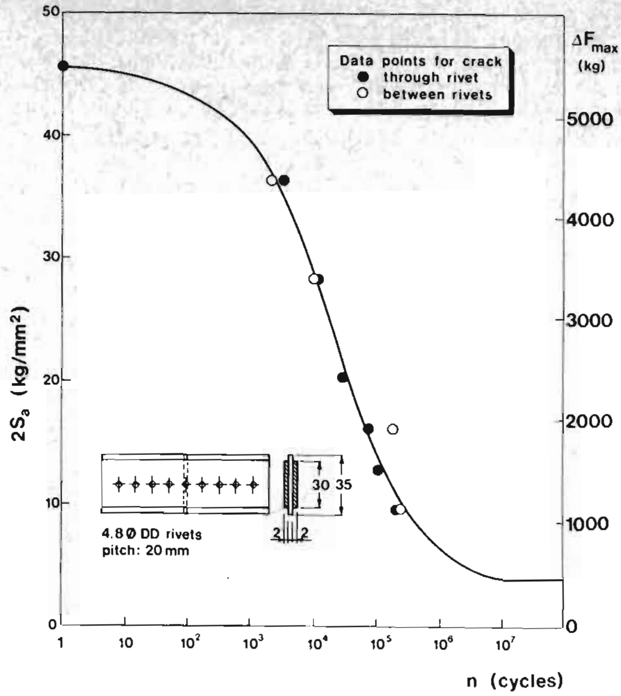


Fig.9 S-N curve of riveted 7075-T6 strip stiffener.

4.1 Prediction of fatigue life of stiffened panel

To predict the fatigue life of a stiffened panel the behaviour of both sheet and stiffener under alternating stresses has to be considered. The crack propagation curve of the stiffened sheet can be determined, using equation (10), when the dependence of da/dn on a is known. The procedure of computing this curve is presented in Tab.1.for the panel configuration with 5 intact strip stiffeners with a crack starting from a rivet hole under the central stiffener and passing with both tips through rivet holes of the adjacent stiffeners. In computing da/dn use was made of the relation between ΔK and da/dn of the unstiffened sheet (Fig.7). By plotting the inverse of the computed da/dn values versus the crack length, the additional number of cycles required for each stepwise increase in crack length is computed by integrating the area under the curve over the concerning crack length range (Fig.8) The resulting crack propagation curve is plotted in Fig.10. It has to be noted that the effect of the rivet hole on the crack propagation curve was ignored.
The same procedure as outlined above was also applied to the case of a crack starting from a rivet hole but passing with both tips between two rivets of the adjacent stiffeners. These results are also plotted in Fig.10.

Of course the predicted crack propagation curve of the stiffened sheet will be valid only as long as the stiffeners remain intact. For that reason in predicting the fatigue life of the stiffened panel, the fatigue life of the stiffener has also to be accounted for. With increasing crack length the maximum stiffener load (F_{max}) will increase. Having available F_{max} as a function of crack length, and the crack propagation curve of the stiffened sheet, the relation between F_{max} and n can be determined.
Using this load-cycle history in combination with a cumulative damage rule and a representative S-N curve, the fatigue life of the stiffener that remains when a certain crack length in the sheet has been reached, can be determined. The most widely used cumulative damage rule is that of Palmgren-Miner, stating that failure will occur when

$$\sum \frac{n_i}{N_i} = 1 \qquad (11)$$

a (mm)	n at Δa (fig.8) (kcycles)	L for central stiffener (fig.3a)	$\Delta F_{max} = L.\Delta S.A_s$ *) (kg)	Average ΔF_{max} at Δa (kg)	N at $\Delta F_{max.av.}$ (fig.9) kcycles	$n/N.10^2$	$\sum_{a=a_0}^{a} n/N.10^2$	N at ΔF_{max} (= N_a) (fig.9) (kcycles)	$n_a = N_a(.6 - \sum_{a=a_0}^{a} n/N)$ (kcycles)	$n_{fatigue\ life} = n_a + \sum_{a=a_0}^{a} n$ (kcycles)
10		1.075	967				0			
	19.891			1092	245	8.10				
30		1.351	1217				8.10	170	88.2	108.1
	9.583			1323	140	6.85				
50		1.587	1428				14.95	120	54.0	83.5
	11.027			1484	110	10.00				
70		1.710	1540				24.95	98	34.4	74.9
	11.700			1565	94	12.44				
90		1.769	1590				37.39	90	20.4	72.6
	7.985			1613	88	9.07				
110		1.819	1635				46.46	83	11.2	71.4
	10.710			1649	79	13.55				
130		1.849	1663				60.01	79	0	70.9

*) $\Delta S = 7.5$ kg/mm^2, $A_s = 120$ mm^2

Table 2 Example of calculation of fatigue life of central stiffener of panel with five intact strip stiffeners.

where n_i is the number of cycles applied at a stress level S_i and N_i is the numver of cycles required to cause failure at S_i. However, in the literature results of tests on notched specimens can be found that show a fairly large variation in $\sum n/N$ values. Values smaller and larger than unity appear to occur depending on the load history parameters. A review of the effect of the different parameters on the $\sum n/N$ value can be found in Ref.6. Because of this, the right hand side of equation (11) will be replaced here by a constant A, the value of A being dependent on the parameters mentioned in Ref.6. When a certain crack length (2a) has been reached in the sheet the remaining fatigue life of the stiffener at that crack length, n_a, can be predicted then (assuming that the crack would not grow any further) from

$$\sum_{a=a_o}^{a} \frac{n}{N} + \frac{n_a}{N_a} = A \tag{12}$$

where the first term of this equation represents the stiffener life fraction consumed during crack growth in the sheet from $2a_o$ to 2a. N_a is the number of cycles required to produce stiffener failure at the stress level being present in the stiffener at the crack length 2a. By applying this procedure for different crack lengths, the dependence of the

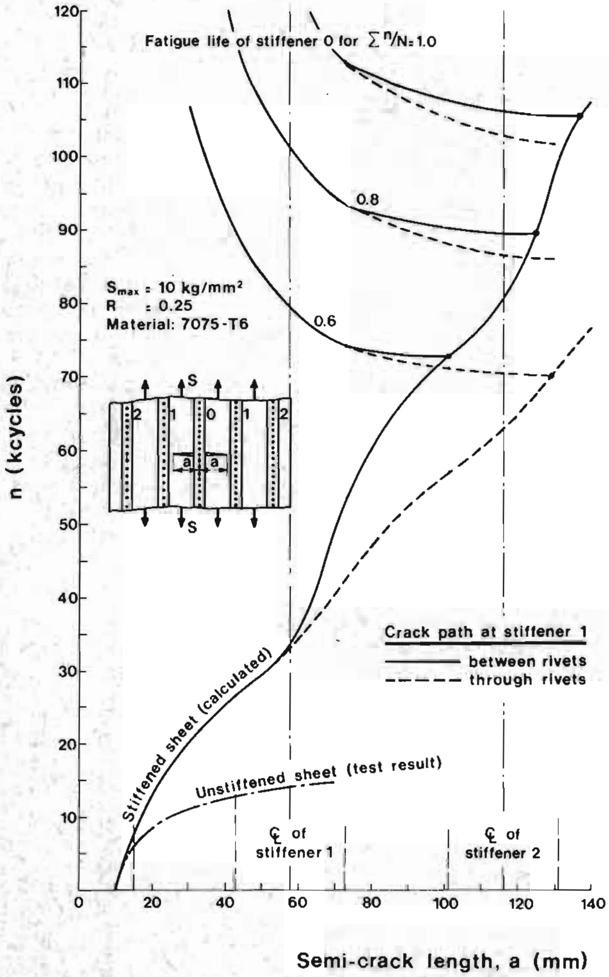


Fig.10 Predicted crack propagation in sheet and fatigue life of central stiffener.

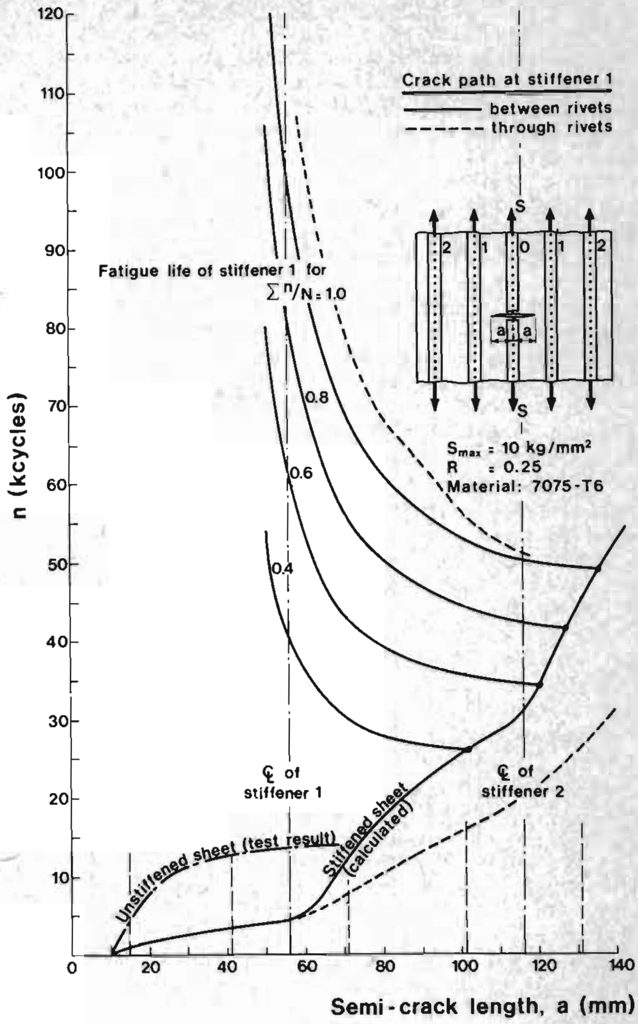


Fig.11 Predicted crack propagation in sheet and fatigue life of stiffener 1.

stiffener fatigue life (being $n_a + \sum_{a=a_o}^{a} n$) on the crack length can be obtained.

An example of the calculation of the stiffener fatigue life as a function of crack length is given in Tab.2. The example applies to the central stiffener of a panel with five intact stiffeners and a central crack. The crack was assumed to pass through rivet holes. For the relation between n and a the calculated results of Fig.8 were used. The S-N curve used was obtained from fatigue tests on loose stiffener specimens (Fig.9). The $\sum n/N$-value (constant A in equation 12) was taken at 0.6.

In Fig.10 the stiffener fatigue life is plotted as a function of crack length for three different $\sum n/N$-values. At a certain crack length the vertical distance between the stiffener fatigue life curve and the sheet crack growth curve represents the remaining fatigue life of the stiffener, n_a. At the intersection of both curves $n_a = 0$. In other words, the stiffener then should fail by fatigue. By applying the same procedure as outlined above the crack propagation curve and the stiffener fatigue life curves were also determined for a panel configuration with 5 stiffeners but with a broken central stiffener. These results are plotted in Fig.11.

5. Experiments

A series of residual strength and crack propagation tests was carried out, using stiffened panels consisting of 2 mm skin stiffened by five flat strip-stiffeners, or by Z-stringers folded from sheet material. The cross-sectional area of the strip stiffeners was equal to that of the Z-stringers. For all panels the total stiffener area was approximately 50 per cent of the total gross area. The panels provided with flat strips were stiffened on both sides to avoid eccentricities. Panels with either an intact or a broken central stiffener were tested. In all cases the stiffeners were riveted to the sheet. Detailed information on the specimens is given in Fig.12.

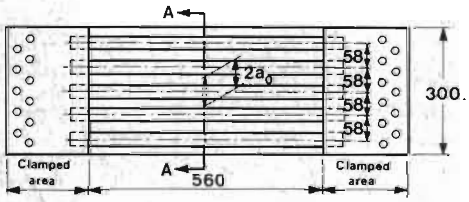
Furthermore, tests were carried out on unstiffened panels to determine the residual strength and fatigue crack propagation characteristics of the unstiffened sheet material (see Figs 6a and 7, respectively). These specimens were cut from the same sheet as used for the stiffened panels.

The material of sheet and stiffeners was 7075-T6 Alclad. The static properties of this material are given in Tab.3.

In all specimens a fine central saw cut was made (originating at a rivet hole) by means of a jeweller's fret saw. It was shown in Ref.7 for aluminium alloy sheet, that this saw cut can simulate a fatigue crack for the purpose of residual strength tests. The saw cut was made before riveting the central stiffener.

Both the residual strength tests and the crack propagation tests were carried out in a 50-tons Amsler hydraulic testing machine, suitable for both static and dynamic testing. Load was applied to the specimen by bolting the specimen ends to a clamping device which was mounted in the testing machine by means of a pin. Initial bending of the panels was prevented by clamping the specimens in such a way that the line of action of the applied loads passed through the centroid of the undamaged cross-section of the panel.

During the tests crack growth records were made. The crack growth recording during the residual strength tests is described in Ref.8. Fatigue cracking was performed under fluctuating tension at a cycling frequency of 250 cycles per minute. Only constant amplitude tests were carried out. The maximum stress was equal to 10 kg/mm^2 and the stress ratio was R = 0.25. Crack propagation observations were made by means of a magnifying glass. The crack growth records were started at a total crack length of 20 mm. The number of cycles was recorded each time the crack passed one of the line markings engraved in the specimen surface at a spacing of 5 mm



Material of sheet and stiffeners: 7075-T6
Rivets: 4.8 mm DD, Universal head
Rivet pitch: 20 mm
Sheet thickness: 2 mm

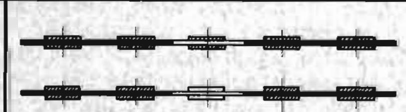
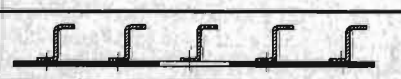
Panel type	Cross-section A-A	Type of stiffener
A B	 Broken central stiffener	Strip stiffener Width 30 mm Thickness 2 mm each
C		Z-stiffener $A_s = 120mm^2$

Fig.12 Dimensions (mm) and detailed geometry of specimens.

Material	Thickness (mm)	$\sigma_{0.2}$ (kg/mm^2)	σ_{ult} (kg/mm^2)	δ% 50mm	Material used for
7075-T6 Alclad	2.0	$50.7^{+1.2}_{-1.9}$	$55.6^{+0.8}_{0.7}$	15	Sheet of unstiff. and stiff. panels
		$52.2^{+0.4}_{-0.4}$	$56.8^{+0.2}_{0.2}$	12	Strip stiffeners
	1.6	$48.5^{+0.4}_{-0.5}$	$53.6^{+0.8}_{0.7}$	14	Z-stiffeners

Table 3 Static properties of sheet and stiffener material (from tensile tests).

6. Test results

6.1 Residual strength tests

The crack growth histories of the various stiffened panel configurations tested are plotted in Figs 13 through 15. It is noted that in the case of the panels with symmetric strip stiffeners, the part of the crack path under the stiffeners could not be observed. It is assumed here that the crack at unstability ran directly into a rivet hole and was arrested there until final failure of the panel. This is indicated in the relevant figures by a vertical portion in the crack growth history at a semi-crack length equal to the stiffener pitch plus half the rivet diameter.

For each value of the initial crack length the stress level at which total failure of the panel occurred is indicated in Figs 13-15. On the basis of these results the flat parts in the residual strength diagrams of the various panel configurations were determined.

6.2 Crack propagation tests

The crack propagation in the sheet of the stiffened panels is plotted in Figs 16 through 18. Crack growth values are plotted for each crack tip separately, except for those cases where the difference in semi-crack length for both crack tips amounted less than 5 mm. In those cases the average for both crack tips is plotted. Only test results are plotted of panels where the crack propagated with both tips in the same way across a rivet line (i.e. either through a rivet hole or between rivets). No crack growth recording could be obtained when the crack tip propagated under the stiffeners. Therefore the dotted curves in this region are only approximate. In the case the crack path ran through a rivet hole it could not be determined exactly how long the crack was arrested in the rivet hole.

The tests were discontinued when one of the stiffeners failed.

7. Discussion

7.1 Residual strength of stiffened panels

The test results of cracked stiffened panels (Figs 13 through 15) provide conclusive evidence in support of the analysis presented in section 3.2 concerning the behaviour of stiffened panels with increasing external load. Apparently there are two failure modes depending on the length of the initial crack as compared with the stiffener pitch:

(i) panels with short crack lengths behave essentially in the same way as the unstiffened panel. Unstable crack growth will result in immediate total failure of the panel (see specimens 8 and 9 in Fig.13).

(ii) panels with longer cracks will behave entirely different. At fracture instability, after some unstable growth, the crack will be arrested under the adjacent stiffeners. After crack arrest the external load can be further increased until the panel fractures completely. The failure load is then essentially independent of the initial crack length. Relating the failure load to the initial crack length a horizontal level in the residual strength diagram is obtained (see specimens 1 and 2 in Fig.13).

The foregoing implies that the horizontal level in the residual strength diagram (denoted by $\bar{\sigma}$) will constitute a lower bound of the residual strength for initial crack lengths that are smaller than the stiffener pitch. Further, if instability occurs for any combination of crack length and stress below $\bar{\sigma}$ then the unstable crack will be arrested, thus offering the possibility to be detected at the next inspection. Hence, $\bar{\sigma}$ can be considered as the fail-safe stress level of the stiffened panel.

It has to be emphasized here that it is not essential for crack arrest that the crack runs into a rivet hole. Crack arrest is basically a result of the reduction of the tip stress intensity due to load transmittal to the stiffener. In Ref.2 results

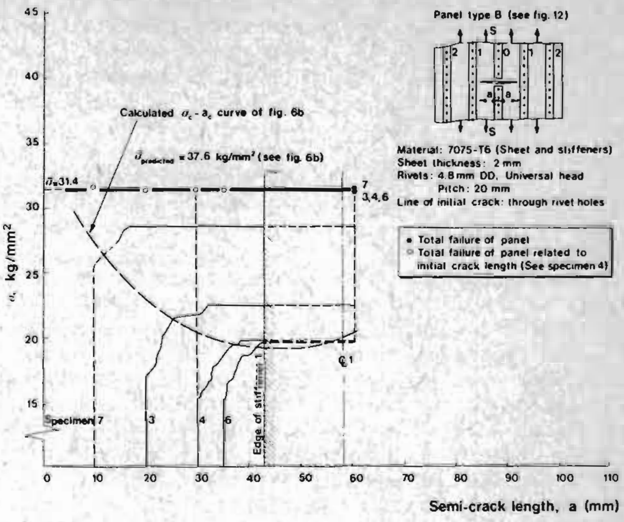


Fig.14 Results of residual strength tests on panels with five strip stiffeners, central stiffener broken.

of residual strength tests are given of panels showing crack arrest between rivet holes. With increasing load after crack arrest the crack propagated stably between the rivet holes until final failure of the panel. However, the stress level $\bar{\sigma}$ was lower as compared to the same panel configuration with the crack path running through the rivet holes (see also Fig.6).

In Figs 13 and 14 the calculated σ_c - a_c curves of Fig.6 are plotted also. Apparently the instability points obtained from tests fit in fairly well with these curves.

Comparing the predicted $\bar{\sigma}$ -levels of Fig.6 with those obtained from tests there appears to be good agreement for the panel configuration with an intact central stiffener (see Fig.13), whereas the residual strength of the panel with a broken central stiffener was overestimated (see Fig.14). It has to be remarked, however, that the predicted $\bar{\sigma}$ -

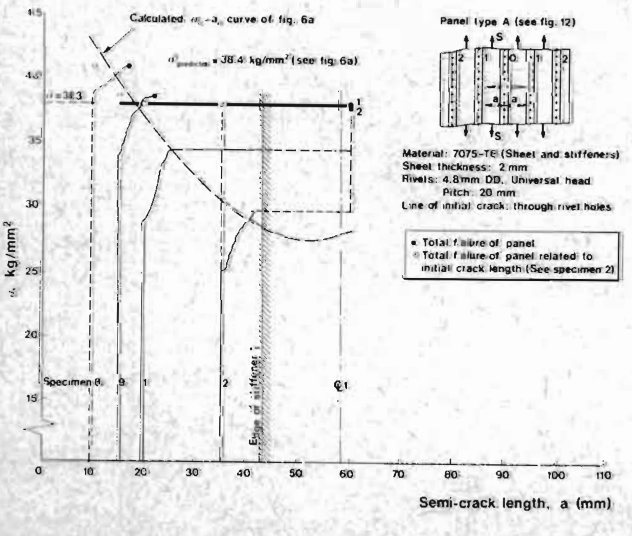


Fig.13 Results of residual strength tests on panels with five intact strip stiffeners.

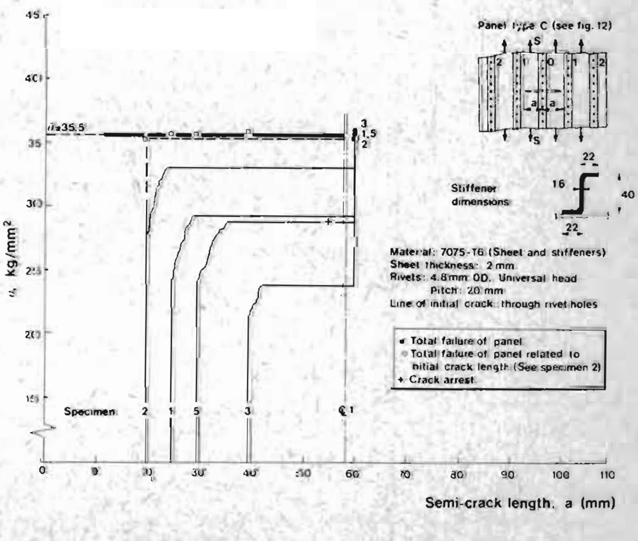


Fig.15 Results of residual strength tests on panels with five intact Z-stiffeners.

levels were determined assuming that the crack, after crack arrest, would remain in the rivet hole until final failure of the panel due to stiffener failure (see Fig.6). If, however, this assumption does not hold an accurate prediction of the $\bar{\sigma}$ -level cannot be made. No stiffened panel data are available for reinitiation of crack growth from the rivet hole. Unfortunately the present test specimens with symmetrical strip stiffeners did not allow an observation of crack behaviour at the rivet hole.

The catastrophic effect of fracture of the central stiffener (for example by fatigue loading) is clearly demonstrated by the results of Figs 13 and 14. The residual strength with an intact central stiffener is at a level of $\bar{\sigma}$ = 38.3 kg/mm^2, whereas in the case of a fractured stiffener failure occurs at stress levels above 31.4 kg/mm^2. Anyway, fracture of the central stiffener will result in an 18 per cent reduction of residual strength.

The effect of the eccentricity of the stiffener on the residual strength can be evaluated by comparing the results of Figs 13 and 15 applying to panels that were similar except for the type of stiffeners. It is observed that the panel configuration with Z-stringers shows only a slightly lower residual strength than the panel with strip stiffeners.

7.2 Fatigue crack growth and fatigue life of stiffened panels

Comparing the crack propagation test results with the calculated propagation curves (Figs 16 through 18) there appears to be a fairly good

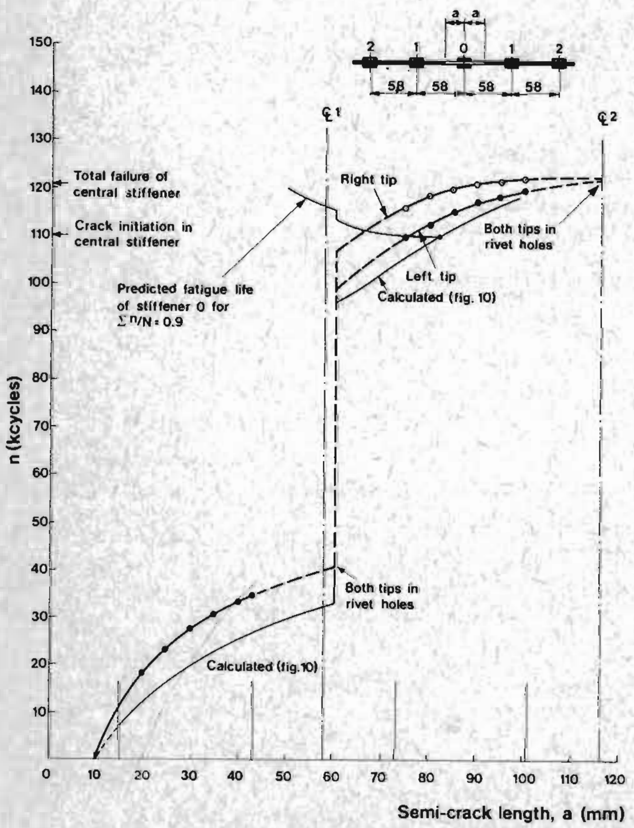


Fig.16 Results of crack propagation test on panel with five intact strip stiffeners (a_o = 10 mm, crack through rivet hole).

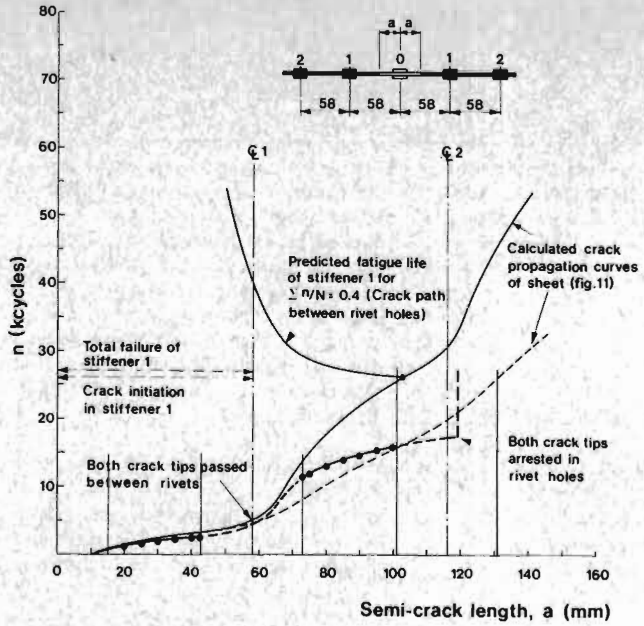


Fig.17 Results of crack propagation test on panel with five strip stiffeners, central stiffener broken (a_o = 10 mm, crack through rivet hole).

agreement. It has to be remarked here that in those cases where the crack was arrested in a rivet hole, the predicted curve was adjusted by partly shifting it upwards. The amount of shifting was determined from the test results. Comparing the arrest time at stiffener 1 of Figs 16 and 18 there appears to be a marked difference (approximately 61 kilocycles in Fig.16 against 17 kilocycles in Fig.18). The much

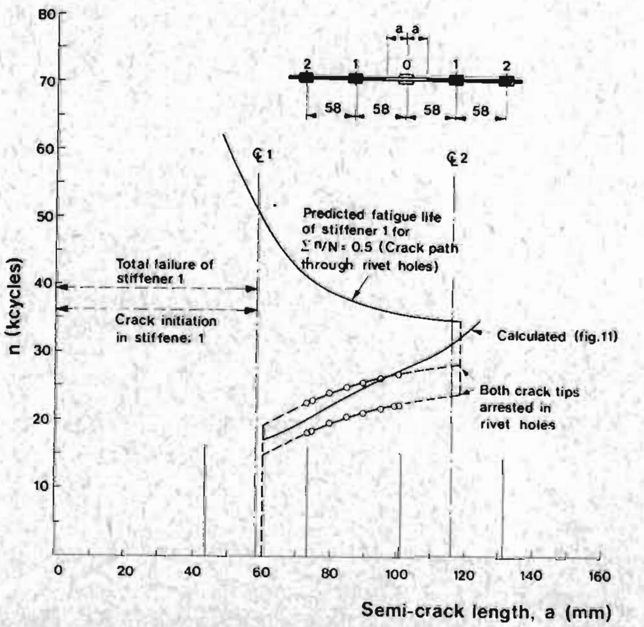


Fig.18 Results of crack propagation test on panel with five strip stiffeners, central stiffener broken (a_o = 60 mm, crack through rivet holes).

lower arrest time of the latter case is apparently due to the presence of the broken central stiffener that exerts extra opening forces on the crack.

In Figs 16 through 18 at the relevant stiffener the number of kilocycles is indicated at which crack initiation and total failure occurred. Further the predicted fatigue life curves of the stiffeners were drawn for that specific $\sum n/N$-value that showed best agreement with the test results. It has to be remarked that in determining these curves the number of kilocycles that the crack tips were arrested in the rivet holes was accounted for. Apparently in all cases $\sum n/N$-values smaller than unity apply, the lowest values occurring in the panel configuration with the broken central stiffener. This might be explained by considering the stress amplitude S_a in the relevant stiffeners as a function of the number of cycles (Fig.19). Apparently the stress increase in the central stiffener occurs more gradually than in the edge stiffeners of the panel with a broken central stiffener. Further the stress in the central stiffener is constant during the greater part of its life. Because of the fact that the stress history of the central stiffener is almost similar to constant amplitude loading (where $\sum n/N$ is exactly equal to unity), a higher $\sum n/N$-value can be expected for that stiffener.

One final remark has to be made here with respect to the computation of the stiffener fatigue life curves. In determining these curves use was made of an S-N curve obtained from tests on loose stiffener specimens. In these specimens the load transfer at the first rivet (= rivet closet to crack) will differ from that at the corresponding rivet in the panel. This might imply that the S-N curve thus obtained is not sufficiently representative for the determination of the fatigue life of the stiffeners of the panel. Some further work to clarify this problem is recommended.

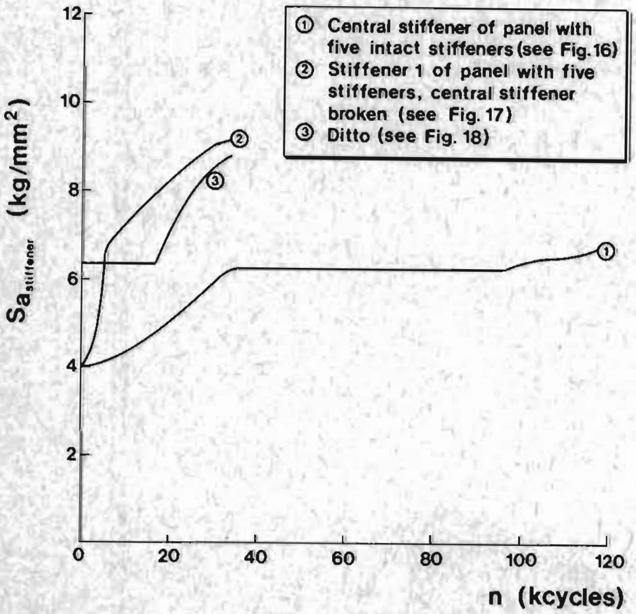


Fig.19 Maximum stress amplitude in critical stiffener during constant amplitude fatigue testing.

7.3 Some remarks concerning fail-safe designing of built-up sheet structures

It was shown already that the establishment of the inspection interval is a very essential part of the fail-safe design. The inspection interval must be chosen so that there is practically 100 per cent certainty of discovery of the crack before it reaches a critical size. In determining the inspection interval the procedure has to start out with the establishment of the required fail-safe strength. This usually corresponds with a load between 80 % and 100 % of the design limit load. Then the critical crack length at this fail-safe strength has to be established by determining the residual strength of the structure for a range of crack sizes (see Fig.1). It was shown in the foregoing that the residual strength diagram of the stiffened panel contains a horizontal portion. The stress level corresponding with this horizontal portion (indicated by $\bar{\sigma}$) is, for a certain amount of damage to be tolerated, determined by the residual strength properties of the unstiffened panel and the sheet-stiffener interaction (expressed by the values of C and L). If at any stress level below $\bar{\sigma}$ fracture instability might occur then the unstably growing crack will be arrested. Thus the stiffened panel can be designed fail-safe by choosing the required fail-safe strength equal to $\bar{\sigma}$. The amount of damage to be tolerated is not exactly specified in any of the requirements of the regulating agencies. The FAA requires that the structure shall be capable of sustaining damage amounting to a single principal structural element when subjected to fail-safe loading. In the case of built-up sheet structures different prescribed maximum damage levels deserve consideration e.g.:

(i) a one-bay skin crack
(ii) a two-bay skin crack with the central member intact
(iii) a two-bay skin crack with the central member failed.

In the past the one-bay panel damage has been adopted by many designers. However, to be realistic one should consider how structural damage in service most frequently occurs. Past experience has shown that the majority of damage incurred in service is due to fatigue. Fatigue cracks often initiate from rivet holes (either in the skin or in the stiffening elements or in both) and grow in both directions Therefore the damages mentioned in (ii) and (iii) are more realistic in fail-safe designing. Having chosen the maximum tolerable crack length for the stiffened panel, for a given inspection technique the time that is available for inspection can in principle be determined when the stiffened sheet crack propagation and the stiffener fatigue life are known. These data can be obtained following the procedure outlined in section 4.1. However, a complication in this connection will be that the fatigue crack propagation in the sheet and the fatigue life of the stiffener for the real structure will have to be based on the pertinent load spectrum of the aircraft. A discussion of the specific problems that will be encountered herewith and some possible means of solution can be found in Ref.9.

It can be concluded from Fig.1 that, for a certain prescribed fail-safe strength, the crack propagation period available for crack detection can be enlarged by selecting sheet material with a higher fracture toughness and better crack propagation properties and last but not least by improvement of the inspection technique. An improvement of the fracture toughness implies an increase of the critical crack length and this will give a longer crack propagation life. However, crack propagation rates in this range of crack lengths are already high implying that only a small gain in available inspection time will be obtained. On the other hand a reduction of the minimum detectable crack length due to an improved inspection technique will give a much larger increase of the crack propagation period available for crack detection because of the much lower crack propagation rate in this range. For the same reason an improvement of the crack propagation properties will be more important than an improvement of the fracture toughness.

A final remark seems in place with regard to the importance of the horizontal level, $\overline{\sigma}$, in the residual strength diagram. This level represents the static residual strength of a structure in which a certain amount of damage has developed (see (i) through (iii) above). This damage may have been caused by stable fatigue crack extension entirely or partly by unstable crack growth under a high load excursion followed by arrest. When the structure for example contains a damage specified in (ii), it should be realized that the fatigue characteristics of a structure with a damage of that size are not spectacular. The central stiffener will be fatigue prone due to a high load concentration factor. If it fails the static residual strength is significantly reduced immediately (compare Figs 13 and 14), while at the same time the remaining fatigue life of the adjacent stiffeners will be reduced (see Fig.18). Also, further growth of the arrested crack may start to reduce the residual strength of the structure. Hence, at the moment that a two-bay skin crack has developed a degradation of the residual strength below the $\overline{\sigma}$-level corresponding with that damage is imminent. If this situation occurs due to a peak load shortly after an inspection the deterioration of strength during that interval may be quite spectacular. This fact emphasizes the importance of early detection of skin cracks and, in particular, of stiffener cracks.

8. Conclusions

The present paper presents results of analytical computations of residual strength and fatigue crack propagation characteristics of built-up sheet structures, using the strength and fatigue properties of sheet and stiffener separately and accounting for sheet-stiffener interaction. The computational results are compared with test results. These tests were performed on riveted specimens consisting of a 2 mm flat sheet stiffened by symmetric strip stiffeners or eccentric Z-stringers. The material of sheet and stiffeners was 7075-T6.

The following conclusions can be drawn from the calculations and the test results:

(i) in a fail-safe design, apart from a reliable inspection technique, a thorough knowledge of the residual strength and fatigue crack propagation characteristics of the structure is required. These characteristics appear to be fairly well predictable by means of computations, when the strength and fatigue properties of sheet and stiffener, separately, are available.

(ii) an improvement of the fail-safe characteristics of a certain design can be obtained, in order of importance, by improving the inspection technique and by selecting material with better crack propagation properties or a higher fracture toughness.

(iii) in a proper fail-safe design the stiffeners can act as crack stoppers in the case of fracture instability. For crack arrest it is not essential that the crack runs into a rivet hole. However, it is considered advantageous to increase the chance of crack arrest in rivet holes by suitable design of rivet patterns because of the better residual strength and stiffener fatigue life properties in that case.

(iv) a skin crack that extends across a stiffener may cause failure of that stiffener due to fatigue. Failure of the stiffener will reduce the residual strength considerably (compare Figs 13 and 14). Further, in the case the skin crack has extended across two bays and is arrested in rivet holes, a much shorter arrest time will result as compared to the case with an intact central stiffener (compare Figs 16 and 18), leading to even worse residual strength properties (see Fig.6b). Therefore failure of a stiffener that extends across a skin crack can easily lead to a dangerous situation.

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